

# Mathematics

## **ALGEBRA**

Groups, subgroups, normal subgroups, quotient groups, homomorphisms, cyclic groups, permutation groups, Cayley's Theorem, Fundamental Theorem of homomorphism, Isomorphism theorems, class equation, Sylow theorems and their applications; Rings, ideals, prime and maximal ideals, quotient rings, unique factorization domain, principal ideal domain, Euclidean domain, Polynomial rings, Fields, finite fields, field extensions.

## **LINEAR ALGEBRA**

Vector spaces, subspaces, Linear dependence, basis and dimension, quotient spaces, Linear transformations, Algebra of linear transformations, kernel, range, isomorphism, Matrix representation of a linear transformation, change of bases, linear functionals, dual spaces, rank, system of linear equations, eigen values and eigen vectors, Cayley – Hamilton Theorem, diagonalization, Hermitian, Skew- Hermitian and Unitary matrices, Finite dimensional inner product spaces, Gram Schmidt-Quadratic forms, reduction and classification of quadratic forms.

## **ANALYSIS**

Finite, countable and uncountable sets, Real number system as a complete field, Archimedean property, Sequence and series of functions, uniform convergence, Riemann integral, improper integrals, monotonic functions, types of discontinuity, functions of bounded variations, Lebesgue measure, measurable functions, Lebesgue integral, functions of several variables, directional derivative, partial derivative and total derivative, maxima and minima, Elements of metric spaces, convergence, continuity, uniform continuity, compactness, connectedness, completeness.

## **COMPLEX ANALYSIS**

Analytic functions, Cauchy - Riemann equations, line integrals, Cauchy theorem, Morera's theorem, Liouville's theorem, Cauchy's Integral formula, zeros of analytic functions, Taylor series, Laurent series, Calculus of residues, contour integration, conformal mappings, Mobius transformations.

## **FUNCTIONAL ANALYSIS**

Normed linear spaces, Banach spaces, basic examples of Banach spaces including  $l_p$ ,  $l_p$  ( $1 < p < \infty$ ), Continuous linear operators on normed linear spaces, open mapping theorem, closed graph theorem, Hahn- Banach theorem, Definition and examples of Hilbert spaces, Orthogonal complement of a subspace in a Hilbert space, Orthogonal basis, Gram-Schmidt process, Bessel's inequality, Riesz representation theorem.

## **TOPOLOGY**

Basic concepts of Topology, Continuity, convergence, Homeomorphism, connectedness, compactness, countability, separation axioms, subspaces, product spaces, quotient spaces, Tychonoff's theorem, Urysohn's Metrization theorem.

## **PROBABILITY AND STATISTICS**

Variance, co-variance, standard deviation, Sample space, Discrete probability, Independent events, Bayes theorem. Random variables and distribution functions (Univariate and Multivariate); Expectation and Moments. Independent random variables, Marginal and conditional distributions. Characteristic functions. Probability inequalities (Chebyshev,

Markov, Jensen). Modes of convergence, Weak and Strong laws of large numbers, Central Limit theorems (i.i.d. case).

### **DIFFERENTIAL EQUATIONS**

Existence and uniqueness of solutions of initial value problems for first order ordinary differential equations, singular solutions of first order ordinary differential equations, general theory of homogeneous and non-homogenous linear ordinary differential equations, variation of parameters, Lagrange and Charpit methods for solving first order partial differential equations, general solution of higher order partial differential equations with constant coefficients, classification of second order partial differential equations, method of separation of variables for Laplace, Heat and Wave equations.

### **CALCULUS OF VARIATIONS AND LINEAR INTEGRAL EQUATIONS**

Linear functionals, variation of a functional, Necessary and sufficient conditions for extrema, Euler-Lagrange equation, Linear integral equations of Fredholm and Volterra type, solution by successive substitutions and successive approximations, solution of equations with separable Kernels.

### **LINEAR PROGRAMMING**

Linear programming problem and its formulation, solution of a linear programming problem by graphical method, basic feasible solutions, solution of linear programming problem by simplex method, M-technique, Two-phase method, Dual problem and duality theorem, convex set theory, basic feasible solution of a linear programming problem at vertices of feasible region.

### **NUMERICAL ANALYSIS**

Finite difference, Interpolation equal and unequal intervals, Numerical solutions of algebraic equations, Fixed point iteration methods, and Newton-Raphson method, rate of convergence, Solution of systems of linear algebraic equations using Gauss elimination and Gauss-Seidel methods, Finite differences, Lagrange, Hermite and spline interpolation, Numerical differentiation and integration, Numerical solutions of ODEs using Picard, Euler, modified Euler and Runge-Kutta methods.